Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Final Exam for MAT 2377 3X (Spring 2011)
Probability and Statistics for Engineers.

Time : 3 hours **Professor : G. Lamothe**
Date : July 21, 2011

Name : _____

Student Number : _____

Calculators are permitted. It is an open book exam.
There are 4 short answer questions and 14 multiple choice questions.
The exam will be marked on a total of 30 points.

Submit your answers for the multiple choice questions in the following table.

Question	Answer	Question	Answer
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7		14	

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Short Answer Questions

[4] 1. Material manufactured continuously before being cut and wound into large rolls must be monitored for thickness (caliper). A sample of ten measurements on paper, in mm, yielded (displayed in ascending order)

$$29.6, 30.3, 30.4, 30.5, 30.7, 31, 31.2, 31.2, 32, 32.2.$$

We can summarize the data with the following two sums :

$$\sum x_i = 309.1 \quad \text{and} \quad \sum x_i^2 = 9,559.87.$$

(a) Compute the mean, the median and the quartiles.

(b) Compute the inter-quartile range and the sample variance.

(c) Are there any outliers ?

(Question 1 cont.)

[**4**] 2. If the probability density function of a random variable X is

$$f_X(x) = k\,x^3, \quad 0 < x < 1.$$

(a) Find the value of k.

(b) Compute the probability that X will be between $1/4$ and $3/4$.

(c) Compute the probability that X will be larger than $2/3$.

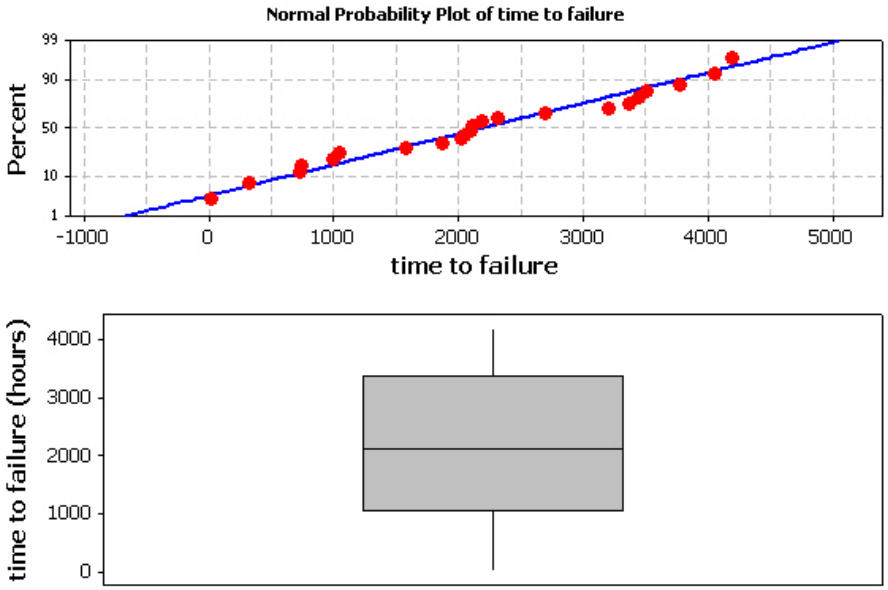
(d) Compute the expectation of X.

(Question 2 cont.)

[4] 3. Consider a random sample of $n = 22$ transformers. For each transformer, we recorded the operating hours until failure. Using R-commander, we produced the following summary table.

mean	sd	0%	25%	50%	75%	100%	n
2191.773	1228.25	10	1177	2103	3320	4186	22

(a) Based on the following normal probability plot and box plot of the 22 times to failure, would you find it reasonable to model the time to failure with a normal distribution ? (Discuss)



(b) Assuming that the time to failure is normally distributed, construct a 95% confidence interval for the true mean time to failure.

(c) Assuming that the time to failure is normally distributed, give the p-value to test that $H_0 : \mu = 2500$ against $H_1 : \mu < 2500$.

(d) Assuming that the time to failure is normally distributed, is there sufficient evidence to conclude that the true mean time to failure is less than 2500 hours at $\alpha = 5\%$?

(Question 3 cont.)

(Question 3 cont.)

[**4**] 4. A large company has many plant shutdowns. Suppose that we can model the shutdowns as a Poisson process with a rate of 2 shutdowns per day.

(a) Give the probability of no shutdowns in 5 days.

(b) Let X the waiting time (in days) for 5 shutdowns. Give the distribution of X with its mean and variance.

(c) Consider a random sample of $n = 50$ waiting times for 5 shutdowns. Approximate the probability that the average of these 50 waiting times will be less than 3 days.

(Question 4 cont.)

Multiple Choice Questions

Submit your answers for the multiple choice questions in the table found on the front page.

[**1**] 1. A research worker wants to determine the average time it takes a mechanic to rotate the tires of a car, and she wants to be able to assert with 95% confidence that the mean of her sample is off by at most 0.5 minute. If she can presume from past experience that $\sigma = 1.6$ minutes, how large a sample .will she have to take ?

(A) 45 (B) 39 (C) 40 (D) 25 (E) 30

[**1**] 2. If the probability that a fluorescent light has a useful life of at least 500 hours is 0.9, find the probability that among 20 such lights at most 18 will have a useful life of at least 500 hours.

(A) 0.2852 (B) 0.3231 (C) 0.9 (D) 0.1501 (E) 0.6083

[**1**] 3. Consider a random sample $X_1, \ldots, X_{15}$ from a normal population with mean $\mu = 10$. The sample mean and the sample variance are $\overline{X}$ and S^2, respectively. Find c such that

$$P\left(\frac{\overline{X} - 10}{S/\sqrt{15}} < c\right) = 0.05.$$

(A) 1.761 (B) 1.645 (C) -1.761 (D) -1.753 (E) -1.645

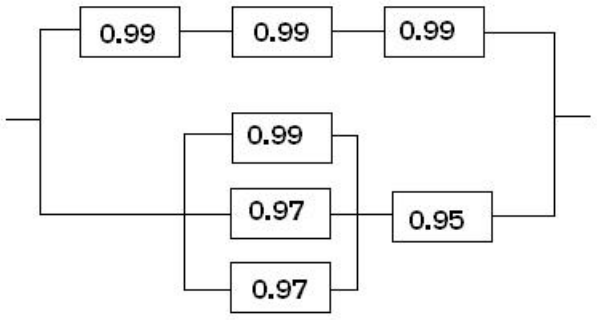
[**1**] 4. A random sample of $n = 50$ suspension helmets used by motorcycle riders and automobile race-car drivers was subjected to an impact test, and on 18 of these helmets some damage was observed. Construct a 95% confidence interval for the true proportion of helmets of this type that would show damage from this test.

(A) $[0.227, 0.493]$ (B) $[0.227, 0.502]$ (C) 0.36
(D) $[0.306, 0.414]$ (E) $[0.305, \infty)$

[**1**] 5. A manufacturer packages boxes with a 12-ounce label weight. Suppose that the actual distribution is $N(12.2, 0.0036)$. What is the probability that a box will weigh less than 12 ounces ?

(A) 0.005 (B) 0.0004 (C) 0.5 (D) 0.0001 (E) 0.25

[**1**] 6. The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates ?



(A) 0.9744 (B) 0.9985 (C) 0.9898 (D) 0.7567 (E) 0.0222

[**1**] 7. The data from 200 machined parts are summarized as follows :

	depth of bore	
edge condition	above target	below target
coarse	15	10
moderate	25	20
smooth	50	80

We select one of the 200 parts at random. Given that the part has a coarse edge condition, what is the probability that the depth of bore is above target ?

(A) 10/25 (B) 15/25 (C) 15/200 (D) 15/90 (E) 75/90

[**1**] 8. Consider the data from Question 7. We select one of the 200 parts at random. Define the events : A =“moderate edge condition” and B =“depth of bore is below target”. Are the events A and B independent ?

(A) Yes (B) No (C) Insufficient Information

[**1**] 9. An engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is normally distributed with $\sigma = 57$ psi. A random sample of 25 specimens is selected to test $H_0 : \mu = 3500$ against $H_1 : \mu \neq 3500$ at a level of significance of $\alpha = 5\%$. Compute the probability of committing an error of type II if mean strength $\mu = 3470$.

(A) 0.7486 (B) 0.2514 (C) 0.8238 (D) 0.1762 (E) 0.0244

[**1**] 10. Consider the hypothesis test from Question 9. From the $n = 25$ observations, we compute a sample mean of $\overline{x} = 3465$ psi. Compute the p-value and give the conclusion at $\alpha = 5\%$.

(A) 0.0022 (B) 0.0011 (C) 0.0250 (D) 0.0500 (E) 0.1145

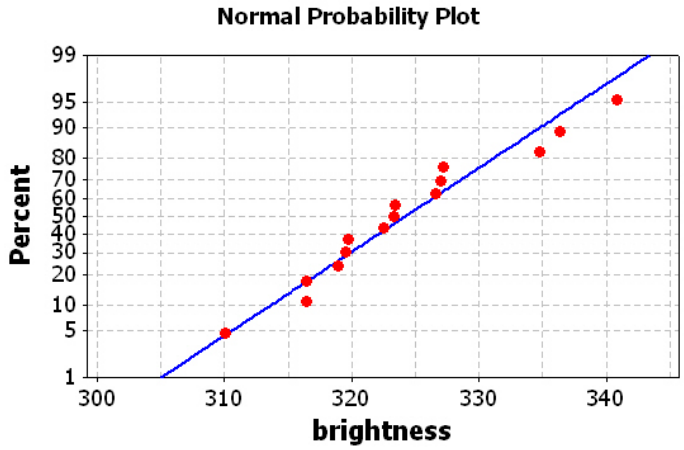
[**1**] 11. The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. The researchers used R-commander to produce the following summary of the data.

mean	sd	0%	25%	50%	75%	100%	n
323.24	9.991482	299.8	317.95	323.3	327.1	340.8	15

Give a point estimate of the population mean and the standard error of the estimate.

(A) point estimate = 323.24 ; standard error = 9.991482
(B) point estimate = 323.24 ; standard error = 9.15
(C) point estimate = 323.24 ; standard error = 2.579790
(D) point estimate = 323.24 ; standard error = 0.666099
(E) point estimate = 323.3 ; standard error = 323.24

[**1**] 12. Consider the data from Question 11. The engineers produced the following normal probability plot.



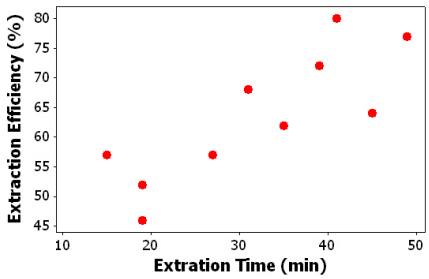
It is reasonable to assume that the brightness is normally distributed. Give a 95% confidence interval for the true brightness level.

(A) [318.18,328.30] (B) [316.71,329.77] (C) [318.08,328.40]
(D) [317.71,328.77] (E) [317.21,329.27]

[**1**] 13. An assembly plant receives its voltage regulators from three different suppliers, 30% from supplier A, 60% from supplier B and 10% from supplier C. Suppose that 95% of the voltage regulators from supplier A perform according to specifications and that the rates for suppliers B and C are 80% and 65%, respectively. If a voltage regulator perform according to specifications, what is the probability that it came from supplier A.

(A) 0.1226 (B) 0.4545 (C) 0.9500 (D) 0.3000 (E) 0.3434

[**1**] 14. A chemical company, wishing to study the effect of extraction time on the efficiency of an extraction operation, obtained the data displayed in the following scatter plot. The variables are : $y =$ extraction efficiency (in %) and $x =$extration time (in minutes).



Here are marginal summary statistics :

variable	n	mean	variance
x	10	32	138.89
y	10	63.5	119.17

The estimated regression line is $\hat{y} = 39.1 + 0.764\,x$. The sample correlation between x and y is

(A) 0.15 (B) 0.75 (C) 0.82 (D) 0.94 (E) -0.85